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THEORY OF NEUTRON EMISSION IN FISSION

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Neutron emission in fission is usually described in terms of two observables: the energy spectrum of emitted neutrons $N(E)$ and the average number of neutrons emitted per fission, or average neutron multiplicity, $\bar{\nu}_p$. These observables are measured before the residual fission fragments decay toward the valley of β stability and are therefore referred to as the prompt neutron spectrum $N(E)$ and the average prompt neutron multiplicity $\bar{\nu}_p$. They are of fundamental importance to the design of macroscopic systems that are driven by the fission reaction, such as thermal or fast reactors. It is the purpose of this paper to describe existing theoretical models for these two observables. Other observables for neutron emission in fission will not be described here due to space limitations.

In the early days of fission, and up to recent years, two simple representations of $N(E)$ have primarily been used. These are the Maxwellian spectrum containing a single temperature parameter T_M , and the Watt spectrum¹ containing both a temperature parameter T_W and a kinetic energy parameter of the moving fragment, E_f . In each case, the parameters are adjusted to optimally reproduce the experimental spectrum for a given fissioning nucleus at a given excitation energy. At the same time, the variation of $\bar{\nu}_p$ with the energy E_n of the incident neutron inducing the fission has been modeled² by a simple polynomial (usually linear) in E_n for each fission system considered: $\nu_p = \nu_0 + \alpha E_n$, and again the two parameters appearing are adjusted to optimally reproduce the experimental multiplicity. Clearly, a need existed for theoretical approaches by which these observables could be predicted both for unmeasured systems and for the understanding of measured systems.

In recent years two new theoretical approaches addressing this need have been actively pursued. The first of these was begun³ at Los Alamos in 1979 and is based upon standard nuclear evaporation theory.^{4, 5} The second was begun⁶ at Dresden in 1982 and is also based upon nuclear evaporation theory, but accounts for neutron cascade emission.

The Los Alamos theory⁷ addresses both neutron induced and spontaneous fission and accounts for the physical effects of (1) the motion of the fission fragments, (2) the distribution of fragment excitation energy, (3) the energy dependence of the inverse process to neutron emission, and (4) multiple chance fission. In particular, to simulate the initial distribution of fragment excitation energy and subsequent cooling as neutrons are emitted, a triangular approximation to the corresponding fragment residual nuclear temperature distribution is used. The maximum temperature T_m of this distribution is determined from

other known or calculable fission observables. With the inclusion of these physical effects, the prompt fission neutron spectrum for a single fragment is given by

$$N(E, E_f, \sigma_c) = \frac{1}{2\sqrt{E_f T_m^2}} \int_{(\sqrt{E} - \sqrt{E_f})^2}^{(\sqrt{E} + \sqrt{E_f})^2} \sigma_c(\epsilon) \sqrt{\epsilon} \, d\epsilon \int_0^{T_m} k(T) T \exp(-\epsilon/T) \, dT, \quad (1)$$

where E and ϵ are the laboratory and center-of-mass neutron energy, respectively, $\sigma_c(\epsilon)$ is the compound nucleus formation cross section for the inverse process, E_f is the average kinetic energy per nucleon of the moving fragment, T and T_m refer to the residual nuclear temperature distribution, and $k(T)$ is the temperature-dependent normalization constant. The total spectrum $N(E)$ is then obtained by combining the contributions from all participating fragments. When $\sigma_c(\epsilon)$ is assumed constant, Eq. (1) reduces to a four-term closed expression involving the exponential integral and the incomplete gamma function [see Eq. (13) of Ref. 7]. When neutron-induced multiple-chance fission occurs, $N(E)$ is combined with the pre-fission neutron evaporation spectrum for each n 'th-chance fission component. The components are then combined in proportion to the n 'th-chance fission probabilities [see Eq. (53) of Ref. 7].

The average prompt neutron multiplicity is obtained using energy conservation, the average energy $\langle \epsilon \rangle$ of the center-of-mass spectrum, other known or calculable fission observables, and is given by

$$\bar{\nu}_p = \frac{\langle E^* \rangle - \langle E_\gamma^{\text{tot}} \rangle}{\langle S_n \rangle + \langle \epsilon \rangle}, \quad (2)$$

where $\langle E^* \rangle$ is the total average fission-fragment excitation energy, $\langle E_\gamma^{\text{tot}} \rangle$ is the total average prompt gamma energy, and $\langle S_n \rangle$ is the average fission fragment separation energy. For neutron induced fission, the numerator of Eq. (2) depends linearly upon E_n , through $\langle E^* \rangle$, whereas the denominator depends, approximately, upon $\sqrt{E_n}$, through $\langle \epsilon \rangle$. In circumstances of neutron induced multiple chance fission, Eq. (2) is combined with the pre-fission neutrons for each n 'th chance fission component and, as before, the components are combined in proportion to the n 'th chance fission probabilities [see Eq. (57) of Ref. 7].

As an example of the Los Alamos approach, $N(E)$ has been calculated for the neutron induced fission of ^{235}U up through third chance fission.^{8,9} The resulting fission

spectrum matrix $N(E, E_n)$ and ratio matrix $R(E, E_n) = N(E, E_n)/N(E, 0)$ are illustrated in Figs. 1 and 2. The figures show that multiple-chance fission effects induce a *staircase effect* in the peak regions of the spectra and an *oscillatory effect* in the tail regions of the spectra, with increasing E_n . Portions of $N(E, E_n)$, and the corresponding $\bar{\nu}_p(E_n)$, have been compared with experiment and good agreement has been found in these comparisons, which are summarized in Refs. 7-9.

The Dresden theory^{10,11} has been applied to spontaneous fission and accounts for the physical effects of (1) the motion of the fission fragments, (2) the distribution of excitation energy in *individual* fragments, (3) the energy-dependence of the inverse process, (4) the center-of-mass anisotropy, and (5) neutron cascade emission. The approach is applicable to well-studied fissioning systems where the distributions of various required fission observables are experimentally known. With the inclusion of the above physical effects the prompt fission neutron spectrum is given by

$$N(E) = \sum_A \int dTKE P(A, TKE) N(E, A, TKE) , \quad (3)$$

where $P(A, TKE)$ is the fragment yield for a given mass number A and total kinetic energy TKE , and $N(E, A, TKE)$ is the corresponding spectrum, analogous to Eq. (1) but including cascade emission and center-of-mass anisotropy effects. This approach has been used to calculate the spectrum for the $^{252}\text{Cf}(sf)$ standard reaction. Good agreement has been obtained with the evaluation of the spectrum performed by Mannhart.¹²

The many other calculations and comparisons with experiment performed using the two approaches just described provide a basis for the prediction of fission neutron spectra and multiplicities in unmeasured systems with good confidence.

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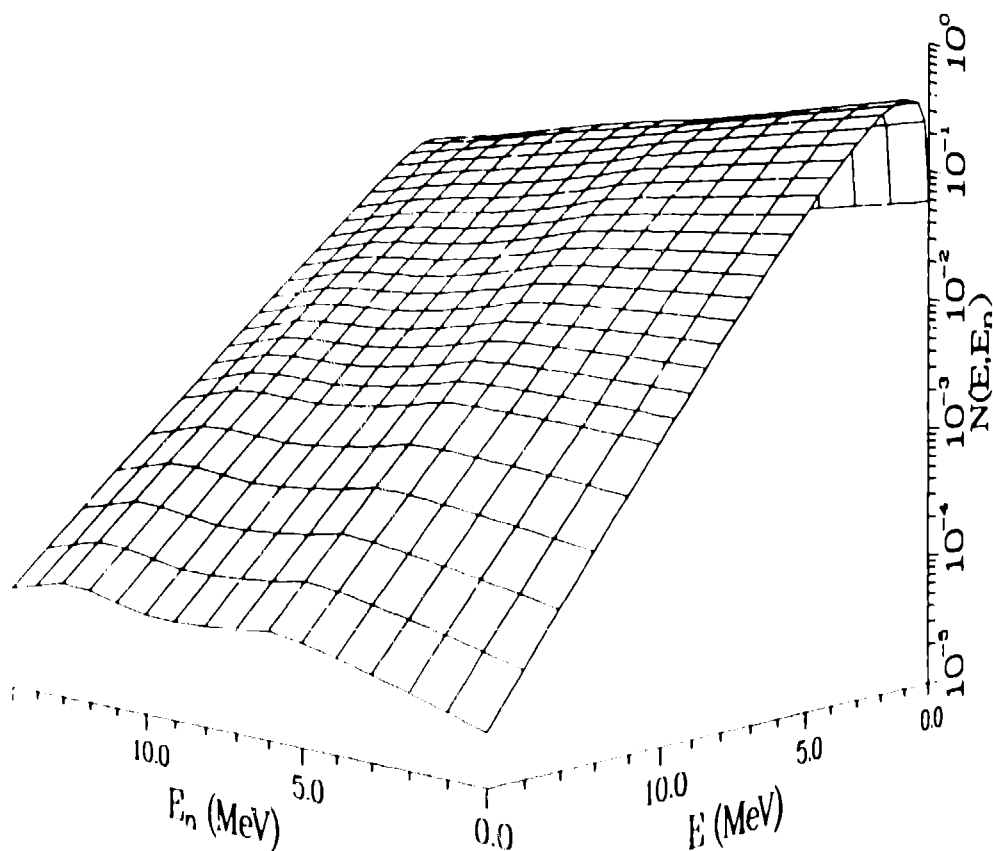


Fig. 1. Prompt fission neutron spectrum matrix $N(E, E_n)$ for the neutron-induced fission of ^{235}U as a function of incident neutron energy E_n and emitted neutron energy E .

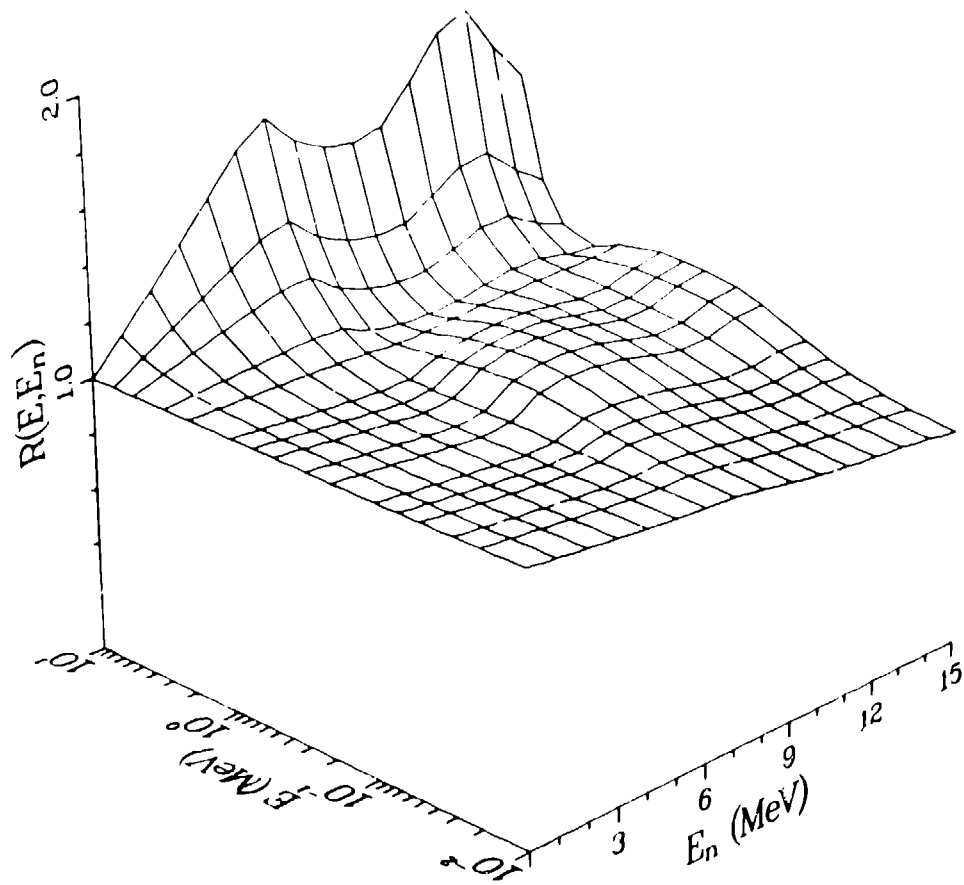


Fig. 2. Prompt fission neutron spectrum ratio matrix $R(E, E_n) = N(E, E_n)/N(E, 0)$ corresponding to the matrix shown in Fig. 1.